Directional Emission from a Microdisk Resonator with a Linear Defect

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Microdisk resonator with a linear defect at some distance away from the circumference is studied theoretically. We demonstrate that the presence of the defect leads to (*i*) enhancement of the output efficiency, and (*ii*) directionality of the outgoing light. The dependence of the radiative losses and of the far-field distribution on the position and orientation of the defect are calculated. The angular dependence of the far field is given by a lorentzian with a width that has a sharp minimum for a certain optimal orientation of the defect line. For this orientation the whispering-gallery mode of a circular resonator is scattered by the extended defect in the direction normal to the disk boundary.

I. INTRODUCTION

The idea to use a microdisk geometry as an alternative to the Fabry-Perot cavity in a resonator design for a semiconductor laser was introduced a decade ago[1]. The advantage of this geometry is that the losses for the whispering-gallery modes of a circular resonator are governed by evanescent leakage and, thus, can be very low. Namely, for a mode with a maximal angular momentum $M = nk_0R$, where the n is the effective refraction index, R is the resonator radius, and k_0 is the wave number of the radiation, the quality factor, Q, with exponential accuracy is given by

$$\ln Q = 2k_0R\left[n\ln\left(n + \sqrt{n^2-1}\,\right) - \sqrt{n^2-1}\,\right]. \tag{1}$$

The value of the effective refraction index is determined by the disk thickness and the indexes of an active and surrounding passive layers. In the pioneering paper Ref. 1 the effective index was $n \approx 2$, while k_0R for the smallest microdisk was ≈ 6. Then Eq. (1) yields $Q \approx 5\cdot10^5$. Experimentally measured values of Q are much smaller, $Q \sim 150$, Ref. 2. The discrepancy is partially due to a prefactor neglected in Eq. (1), but primarily due to the absorption in the active layer[1,3]. With such a high Q-value the lasing threshold for a microdisk resonator is very low. For the same reason the output power is also low, which is not desirable. Another serious drawback of the microdisk geometry is that the angular dependence of the output intensity is $I(\psi) \propto \cos^2(M\psi)$, whereas applications require a directed emission. In order to remedy these drawbacks two proposals were put forward
(*i*) to extract the light out of the resonator by using two parallel disks[4]. The first disk with high Q contains a multiple quantum well structure in which the light is generated. The second passive disk coupled to the laser contains an opening serving as a leakage source. The shape of the opening determines the directionality of the output light.
(*ii*) to couple the light out by introducing either an identation in the form of the "tip of the egg"[2] or corrugation[5] on the *circumference* of the disk.
  A radical solution for increasing the output, and, to a certain extent, directionality, by deforming the shape of the disk[6] seem to devaluate the attempts to extract light from a perfectly circular microdisk. This solution relied on the fact that deformation causes a qualitative change in the light-ray dynamics, so that the whispering-gallery trajectory of a ray becomes unstable. As a result, the ray eventually impinges on the boundary at an angle smaller than the critical angle, $\sin^{-1}(1/n)$. This leads to a refractive escape. The improvement of the directionality of the output light from a wave-chaotic resonator was studied theoretically in a great detail[7,8]. The results of calculations for both "bouncing ball" and "bow-tie" modes and $nk_0R \approx 100$ can be roughly summarized as follows. In each 90°-quadrant the output light is concentrated within total angular interval of about 60° with a strong peak of a width $\sim 30°$ and a large number of narrow satellites[7].
  In the present paper we suggest an alternative approach for improving both the directionality and the output efficiency of a *circular* microdisk. This improvement can be achieved by introducing a properly oriented *linear* defect *away from the circumference*. Proposed geometry is illustrated in Fig. 1. The reason why the linear defect causes directional emission from a microdisk is the following. The field of a whispering-gallery mode "tunnels" *towards* the defect line, which then assumes a role of the secondary source. Since the source is extended, it emits a secondary light beam which is weakly divergent. The divergence is minimal when this secondary light beam is emitted in the radial direction, i.e. in the direction normal to the disk boundary. It is convenient to characterize the position and orientation of the defect by two parameters, namely $r_0 \gg k_0^{-1}$ - radial distance from the edge to the circumference, and d - the minimal distance from the defect line to the disk center. As it will be shown below, the optimal orientation of the defect, for which the direction of the secondary beam is radial, is determined by the condition $d = (R-r_0)/\sqrt{2}$. Under this condition the directionality of the output light is maximal. Below we will demonstrate that, with exponential accuracy, the radiative losses caused by the defect are given by

$$\ln Q = \frac{2^{5/2}}{3} \left(\frac{r_0}{R}\right)^{3/2} (nk_0 R). \tag{2}$$

These losses dominate over the evanescent losses Eq. (1) if $r_0 \ll R$. The angular dependence of the defect-induced emission is a lorentzian, which under the optimal condition $d = (R - r_0)/\sqrt{2}$, has the form

$$I(\psi) = \frac{1}{\left(\psi - \frac{\pi}{4}\right)^2 + 2n^2 \left(\frac{r_0}{R}\right)}, \tag{3}$$

with the width which is also governed by the ratio r_0/R. Note, that although Eqs. (2), (3) apply only for $k_0 r_0 \gg 1$, this ratio can still be quite small as long as $k_0 R$ is large.

The paper is organized as follows. In Sec. 2 we derive Eqs. (2), (3) within the scalar diffraction theory. In Sec. 3 we discuss the limits of applicability of the theory and provide numerical estimates.

II. ANGULAR DEPENDENCE OF THE OUTPUT LIGHT

Neglecting the difference between TE and TM polarizations, the field of a whispering-gallery mode in a microdisk represents a solution of the two-dimensional Helmholtz equation

$$\mathcal{E}_M(\rho, \phi) \propto \cos(M\phi) J_M(nk_0\rho), \tag{4}$$

where ρ and ϕ are polar coordinates, M is the angular momentum, and J_M is the Bessel function. We assume that M is close to the maximal value $nk_0 R$. Then the field Eq. (4) is localized at the boundary $\rho = R$ within a narrow ring of a width $\delta\rho \sim R/(nk_0 R)^{2/3} \ll R$. At smaller ρ the field falls off towards the center of the disk as

$$\mathcal{E}_M \propto \cos(M\phi) \exp\left[-\frac{2^{3/2}}{3M^{1/2}} (nk_0 r)^{3/2}\right], \tag{5}$$

where $r = R - \rho$ is the distance from the boundary.

Within the scalar diffraction theory the emitted field caused by the presence of a defect is determined by the Fresnel-Kirchhoff diffraction integral in which the source is the field $\mathcal{E}_M(\rho, \phi)$ taken at $\rho = \rho(\phi)$, where $\rho(\phi)$ describes the defect profile. In the case of a linear defect (Fig. 1) we have $\rho(\phi) = d/\cos\phi$. It is convenient to introduce instead of ρ a variable x which is the distance along the defect (Fig. 1). The relation between ρ and x is the following

$$\rho = \left[d^2 + \left(\sqrt{(R - r_0)^2 - d^2} - x\right)^2\right]^{1/2} = R - r_0 - x\frac{\sqrt{(R - r_0)^2 - d^2}}{R - r_0}. \tag{6}$$

In the second equality we have used the fact that $x \ll R$. Substituting Eq. (6) into Eq. (5) we obtain

$$\mathcal{E}_M(x) \propto \exp\left[-\frac{2^{3/2}}{3} \left(\frac{r_0}{R}\right)^{3/2} nk_0 R\right] \exp(-ax) \cos[M\phi(x)], \tag{7}$$

where

$$a = 2^{1/2} nk_0 R \left(\frac{r_0}{R^3}\right)^{1/2} \frac{\sqrt{(R - r_0)^2 - d^2}}{R - r_0}. \tag{8}$$

In Eq. (7) we assumed that $x \ll r_0$. Indeed, the relevant values of x are $\sim a^{-1}$. Then the condition $x \ll r_0$ can be rewritten as $r_0 a \sim (nk_0 R)(r_0/R)^{3/2} \gg 1$. We see that this condition is equivalent to the requirement that the asymptotic Eq. (5) is valid at $r = r_0$. The first x-independent factor in Eq. (7) determines the dependence of the output field on the defect position, r_0. The expression Eq. (2) immediately follows from this dependence.

The form of the function $\phi(x)$ in Eq. (7) can be easily established from Fig. 1

$$\tan\phi = \frac{\sqrt{(R - r_0)^2 - d^2} - x}{d}. \tag{9}$$

Now we are in position to write the expression for the intensity of the outgoing light in the direction ψ. It is given by the following integral along the defect

$$I(\psi) \propto \left| \int_0^\infty dx \; e^{-ax} \cos\left[M\phi(x)\right] \int_{-\pi}^{\pi} d\varphi \; \exp\left[ink_0 b(x,\varphi) - ik_0 R\cos(\psi - \varphi)\right] \right|^2. \tag{10}$$

The internal integral over φ is a standard Fresnel-Kirchhoff integral. Parameter b in the exponent is the distance from the source on the defect to the exit point (Fig. 1)

$$b^2(x,\varphi) = R^2 + d^2\cos^2\left[\phi(x)\right] - 2Rd\cos\left[\phi(x)\right]\cos\left[\varphi - \phi(x)\right]. \tag{11}$$

It is convenient to express the distance b directly through x and φ, which can be done using Eq. (9)

$$b^2(x,\varphi) = R^2 + (R - r_0)^2 - 2R\left(d\cos\varphi + \sqrt{(R-r_0)^2 - d^2}\;\sin\varphi\right) + 2x\left(R\sin\varphi - \sqrt{(R-r_0)^2 - d^2}\right) + x^2. \tag{12}$$

Recall now, that the values of x in the integral Eq. (10) are small $x \sim a^{-1} \ll r_0$. It can also be seen from Fig. 1 that the outgoing ray is normal to the boundary when $\cos\varphi = d/(R - r_0)$. This suggests that the difference

$$\delta = \varphi - \cos^{-1}\left(\frac{d}{R - r_0}\right) \tag{13}$$

is a small parameter. In other words, the major contribution to the Fresnel-Kirchhoff integral comes from small $\delta \ll 1$. The integrand in Eq. (10) is a rapidly oscillating function. This allows to expand the phase of the oscillations

$$\Phi(x,\varphi) = M\phi(x) + k_0\left[nb(x,\varphi) - R\cos(\psi - \varphi)\right] \tag{14}$$

in terms of x and δ

$$\Phi(x,\varphi) = A_x x + A_{xx} x^2 + 2A_{x\delta}\,x\delta + A_{\delta\delta}\,\delta^2. \tag{15}$$

As it was already stated in the Introduction, the maximal directionality of the outgoing light is achieved for the position of the defect $d = (R - r_0)/\sqrt{2}$. To demonstrate this, we introduce a dimensionless deviation from the optimal defect position

$$\Delta(d) = \frac{d}{R} - \frac{R - r_0}{\sqrt{2}R}. \tag{16}$$

We will see that the width of the function $I(\psi)$ increases dramatically with Δ. Rather involved but straightforward calculations yield the following expressions for the coefficients in the expansion Eq. (15)

$$A_{x\delta} = \frac{nk_0 R}{2r_0}\left(1 - \frac{4\Delta^2}{1 - 4\Delta^2}\right) \approx \frac{nk_0 R}{2r_0}\left(1 - 4\Delta^2\right), \tag{17}$$

$$A_{\delta\delta} = -k_0 R\left[1 - \frac{nR}{r_0}\left(1 - \frac{4\Delta^2}{1 - 4\Delta^2}\right)\right] \approx -k_0 R\left[1 - \frac{nR}{r_0}\left(1 - 4\Delta^2\right)\right], \tag{18}$$

$$A_{xx} = \frac{nk_0}{4r_0} \quad , \quad A_x = a\frac{\psi - \pi/4}{\delta_\psi}, \tag{19}$$

where the parameter δ_ψ in the expression for A_x is defined as

$$\delta_\psi = n\left(\frac{2r_0}{R}\right)^{1/2}\left[\frac{1 - 4\Delta^2}{1 - 4n^2\Delta^2}\right]^{1/2} \approx n\left(\frac{2r_0}{R}\right)^{1/2}\left[1 + 2(n^2 - 1)\Delta^2\right]. \tag{20}$$

With the use of the expansion Eq. (15), the Fresnel-Kirchhoff integral can be easily evaluated yielding

$$I(\psi) \propto \left| \int_0^\infty dx \; \exp\left[-x\left(a - iA_x\right) + ix^2\left(A_{xx} - \frac{A_{x\delta}^2}{A_{\delta\delta}}\right)\right] \right|^2. \tag{21}$$

The remaining integral over x is of the Fresnel-type. However, it cannot be reduced to the special functions $Ci(u)$ and $Si(u)$, which describe the diffraction from a semi-infinite plane[9]. This is because the linear term in the exponent contains a contribution $-ax$ which is *real*. For this reason, it is convenient to introduce a new variable $z = ax$ in the integral (21). Upon substituting the coefficients (17)-(19) into Eq. (21) we arrive at the final result

$$I(\psi) \propto \left| \int_0^\infty dz \, \exp\left[-z\left(1 - i\frac{\psi - \pi/4}{\delta_\psi} \right) + iz^2 \frac{n+1}{4n^2 k_0 r_0} F(\Delta) \right] \right|^2, \qquad (22)$$

where the function $F(\Delta)$ is defined as

$$F(\Delta) = 1 + \frac{8nR\Delta^2}{(n+1)r_0}. \qquad (23)$$

As in Eqs.(17)-(19), we kept only the leading Δ^2 term in the definition of F. Now we can substantiate the statement that the optimal directionality of the emission is achieved at $\Delta = 0$. Indeed, z^2-term in the exponent of Eq. (22) leads to the broadening and oscillations of the angular dependence, $I(\psi)$. At small Δ we have $F \approx 1$; then the z^2-term is multiplied by a small factor $\sim (k_0 r_0)^{-1} \ll 1$ and, thus, can be neglected. Then we immediately recover the lorentzian Eq. (3). On the other hand, for a general position of the defect we have $\Delta \sim 1$, and $F \sim R/r_0$. Then the z^2-term acquires a much larger coefficient $2R/(nk_0 r_0^2)$, resulting in the loss of the directionality of the output light. This is illustrated in Fig. 2. It is seen that significant broadening and sideback oscillations set in already at small values of Δ. In particular, for $\Delta = 0.3$ the broadening is 60 percent.

III. CONCLUSION

Let us first discuss the validity of the assumptions used in the above calculation
(a) $I(\psi)$ was calculated within the scalar diffraction theory using Fresnel-Kirhhof approach. Note, that for a circular geometry, $I(\psi)$ can be calculated exactly by solving the scalar wave equation and treating defect as a perturbation. Then the expression for $I(\psi)$ is given by a sum over angular momenta of the leaking modes. Fresnel diffraction corresponds to replacing this sum by an integral. The accuracy of such a replacement is determined by the next term in the Poisson expansion, which contains an exponential factor $\exp\left[-2^{3/2}\pi n k_0 (r_0 R)^{1/2} \right]$. Thus, the condition of validity of the Fresnel-Kirhgof approach is $r_0 \gg 1/(k_0^2 R)$, which is not restrictive at all.
(b) According to Eq. (3), the full width at half maximum (FWHM) is equal to $2\delta_\psi = 2n(2r_0/R)^{1/2}$. This equation was derived under the assumption that the defect is located far enough from the circumference of the disk, *i.e.* $r_0 \gg \delta\rho \sim R/(nk_0 R)^{2/3}$. It is possible to derive a more general expression for $I(\psi)$, that is valid for $r_0 \sim \delta\rho$, when the asymptotics Eq. (5) is not yet applicable. Derivation is based on the integral representation of the Bessel function and yields

$$I(\psi) \propto \frac{2}{(\pi\gamma)^{1/2}} \int_0^\infty ds \frac{e^{-\gamma s^2}}{(1+s)^2 + \left(\frac{\psi - \pi/4}{\delta_\psi} \right)^2}, \qquad (24)$$

where the parameter γ is defined as

$$\gamma = 2^{1/2} n k_0 R \left(\frac{r_0}{R} \right)^{3/2}. \qquad (25)$$

It is seen that the condition $r_0 \gg \delta\rho$ corresponds to $\gamma \gg 1$. Then we immediately recover the Lorentzian Eq. (3). At moderate γ, the FWHM is given by $2C(\gamma)\delta_\psi$, where the function $C(\gamma)$ is plotted in Fig. 2, inset. It is seen that within the wide interval $1 \lesssim \gamma \lesssim 10$ the broadening factor $C(\gamma)$ changes very slowly. Then the FWHM can be expressed in terms of γ as $2^{4/3} n C(\gamma) \left(\gamma/nk_0 R \right)^{1/3}$, which is also a slow function of γ. Choosing for concreteness $\gamma = 1$, we find for FWHM the expression $3.35 n/(nk_0 R)^{1/3}$.

We now turn to the numerical estimates. Three types of microdisk semiconductor lasers have been described in the literature so far. The lasers for wavelengths $\lambda \approx 1.5\mu m$ have M-values reported[1-4,10-15] are rather low ($10 \lesssim M \lesssim 70$) and $n \approx 2.5$. For this n and maximal $M = 70$ the FWHM is 116°. Lasers for $\lambda \approx 0.8\mu m$[16-18] have $n \approx 3.1$ and also rather small M ($30 \lesssim M \lesssim 300$). With maximal $M = 300$ we get 89° for FWHM. Nitride-based lasers operating at $\lambda \approx 0.4\mu m$[19-21] have much higher M-values ($200 \lesssim M \lesssim 600$) and $n \approx 2.8$. This yields FWHM of 64°. Microdisk lasers based on non-crystalline materials (polymer[22] and dye solution[23]) have also been reported. For this materials $n \approx 1.8$ is smaller and the values of M (930 in[22] and 3000 in[23]) are high. Both factors tend to narrow $I(\psi)$. Namely, for $M = 1000$ the FWHM of 34° can be achieved.

Let us discuss the physical meaning of the optimal condition, $d = (R - r_0)/\sqrt{2}$. As it is seen from Eq. (7), the phase of the whispering-gallery mode changes along the defect. As the defect plays a role of a source of the outgoing light, this change, $\phi(x)$, is equivalent to the rotation of the line of the constant phase by an angle $\sin^{-1}[d/(R - r_0)]$.

Then, under the optimal condition, the line of the constant phase is perpendicular to the radial line drawn through the edge of the defect (Fig. 1). In other words, under the optimal condition, the defect can be replaced by a constant phase line at distance r_0 from the circumference that is *parallel* to the circumference. Clearly, the angular width of the far field emitted by this line is minimal for this parallel orientation.

Note in conclusion, that in[6] the improvement of the output characteristics of microdisk laser, achieved by introducing the deformation, is due to the fact that when the disk is deformed, the light rays are unable to stay within a whispering-gallery trajectory, and experience refractive escape in course of the chaotic motion[24]. In the present paper we considered a perfectly circular microdisk with a defect. A point-like defect at some distance away from the boundary would be unable to couple out all the whispering-gallery modes, since it will not be able to affect the modes having a node at the defect position. Our main message here is that no whispering-gallery mode can evade an *extended* defect and will be directed out of the resonator as a result of scattering by this defect.
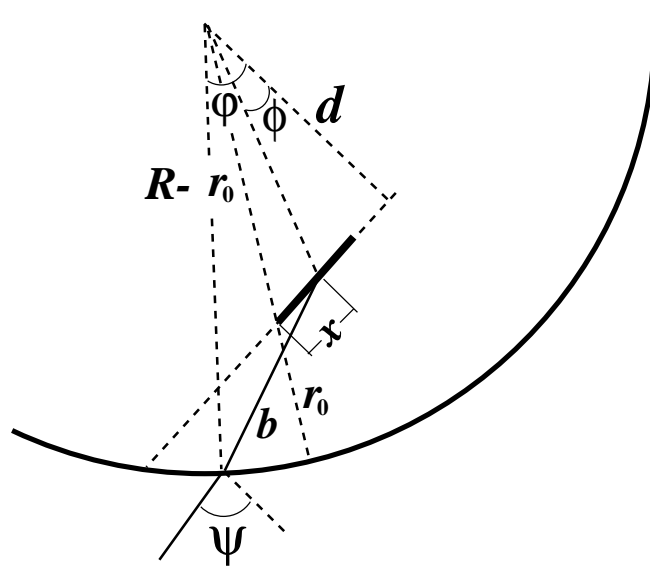

FIG. 1. Shematic illustration of a circular microdisk of a radius R with a linear defect. The defect position is characterized by r_0 - the distance from edge to the disk circumference along the radius; the defect orientation is fixed by the minimal distance from the defect line to the disk center. The direction of the outgoing light is characterized by the angle ψ.
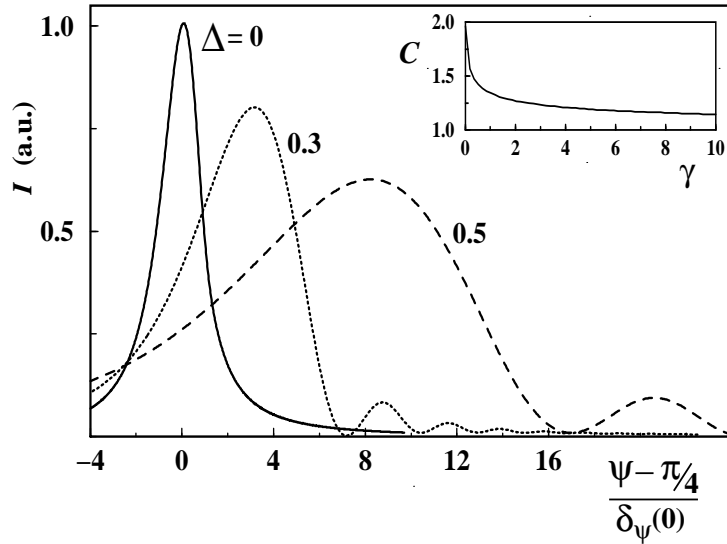

FIG. 2. Angular distribution of the far-field emission intensity is plotted for different deviations Δ (Eq. (16)) from the optimal condition $d = (R - r_0)/\sqrt{2}$. Inset: dimensionless broadening factor C is plotted versus the dimensionless parameter γ, defined by Eq. (25).

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